SOLUTIONS
Mathematics

Section – A

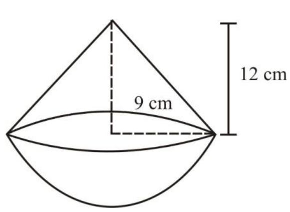
1. For real and equal roots, D = 0
$b^2 - 4ac = 0$
$4(k-1)^2 - 4(k+1) \times 1 = 0$
$(k^2 + 1 - 2k) - k - 1 = 0$
$k^2 - 3k = 0$
$k(k-3) = 0$
$k = 0, k = 3$
$k = 3$ [$\because$ k = 0 neglect]
Or
$b^2 - 4ac = 16 - 4 \times 4 \times 1$
$= 16 - 16$
$= 0$
Roots of the given equation are real & equal.

2. Here, $OA \perp OP$ and $OB \perp PB$ [$\because$Tangent & radius angle]
In ΔPAO, $OP^2 = AP^2 + OA^2 = 15^2 + 8^2 = 225 + 64 = 289$
$\Rightarrow$ OP = 17 cm
In ΔPBO, $PB^2 = OP^2 - OB^2 = 17^2 - 7^2 = 289 - 49 = 240$
$\Rightarrow PB = \sqrt{240} = 4\sqrt{15}$ cm

3. $a_n = a + (n-1)d$
$4 = a + 6 \times (-4)$
$4 = a - 24$
$a = 28$

4. Empirical formula :
Mode = 3 Median – 2 Mean
= 3 (9.6) – 2 (10.5)
= 28.8 – 21.0
= 7.8

5. Radius of the base of cone = $\frac{18}{2}$ = 9 cm



Height of cone (h) = 12 cm
Slant height (l) = $\sqrt{r^2 + h^2}$
$\Rightarrow l = \sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15\ cm$
TSA of toy = CSA of hemisphere + CSA of cone
$= 2\pi r^2 + \pi rl = \pi r(2r + l)$
$= 3.14 \times 9(2 \times 9 + 15)$

$= 3.14 \times 9 \times 33$
$= 932.58 \text{ cm}^2$

6. Let the three consecutive numbers be x, x + 1 and x + 2

$\therefore \quad (x + 1)^2 = (x + 2)^2 - x^2 + 60$

$\Rightarrow \quad x^2 + 1 + 2x = x^2 + 4 + 4x - x^2 + 60$

$\Rightarrow \quad x^2 - 2x - 63 = 0$

$\Rightarrow \quad (x - 9)(x + 7) = 0$

$x = 9$ [Rejecting –7]

Hence, the numbers are 9,1 0 and 11.

Or

Let the usual speed of plane be x km/hr and the increased speed be y km/hr.

$\Rightarrow \quad y = (x + 250)$ km/hr

distance = 1500 km [Given]

Acc. to condition given –

(Scheduled time) – (time taken at increased speed) = 30 min.
= 0.5 hrs.

Also, $\frac{1500}{x} - \frac{1500}{y} = \frac{1}{2}$

$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$ $\left[\because \text{time} = \frac{\text{distance}}{\text{speed}}\right]$

$\Rightarrow$ On solving, we get

$(x - 750)(x + 1000) = 0$

$\Rightarrow \quad x = 750$, or $x = -1000$ (neglect)

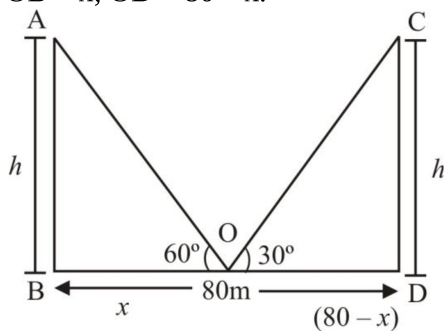
So, the usual speed is 750 km/hr.

Section – B

7. Let BD = width of river = 80m

AB = CD = height of trees = h

OB = x, OD = 80 – x.



In ΔABO, $\tan 60^\circ = \frac{h}{x}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad h = \sqrt{3}x$ ....(1)

In ΔCDO, $\tan 30^\circ = \frac{h}{80 - x}$ ....(2)

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{80 - x}$

On solving (1) & (2)
x = 20 m

So, $h = \sqrt{3} \times 20 = 20\sqrt{3} = 20 \times 1.732 = 34.6\,\text{m}$

Thus, height of trees = 34.6 m
and
Required distances be 20m and 60m.

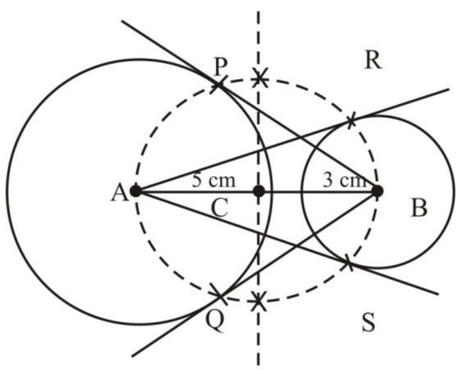
8.

class	*class mark* (x_i)	*frequency* (f_i)	$f_i x_i$
0 – 20	10	16	160
20 – 40	30	14	420
40 – 60	50	24	1200
60 – 80	70	26	1820
80 – 100	90	x	$90x$
		$\Sigma f_i = 80 + x$	$\Sigma f_i x_i = 3600 + 90x$

$$\text{Mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} \Rightarrow 54 = \frac{3600 + 90x}{80 + x}$$

$\Rightarrow$ x = 20.

9.



Steps of constructions :

(1) Draw a line segment AB of 9 cm
(2) Taking A and B as centres draw two circles of radii 5 cm & 3 cm respectively.
(3) Perpendicular bisect the line AB. Let midpoint of AB be C.
(4) Taking C as centre, draw a circle of radius AC which intersects the two circles at point P, Q, R and S.
(5) Joint BP, BQ, AS and AR.
Hence. BP, BQ and AR, AS are the required tangents.

10. Here, Modal class = 40 – 50

$\therefore \quad l = 40, f_1 = \text{p}, f_0 = 12, f_2 = 18$ and $h = 10$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \Rightarrow \qquad 48 = 40 + \frac{p - 12}{2p - 12 - 8} \times 10$$

$$\Rightarrow \quad 8 = \frac{10p - 120}{2p - 30} \quad \Rightarrow \quad 16p - 240 = 10p - 120$$

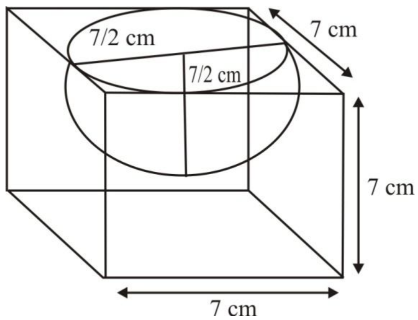
$\Rightarrow \quad 6p = 120$

$\Rightarrow \quad p = 20$

Section – C

11. Diameter of hemisphere = Edge of cube = 7 cm

Radius $= \frac{7}{2}$ cm.



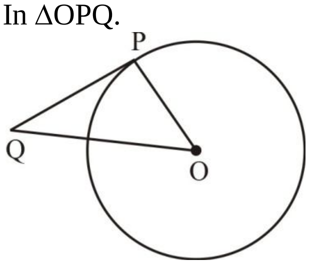
Req. S.A. = SA. of cube – area of top of hemisphere + CSA of hemisphere.

$= 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$

$= 6(7)^2 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = 294 + 38.5$

$= 332.5\ cm^2$

12. In ΔOPQ.



$\angle P + \angle Q + \angle O = 180^o$ [$\angle O = \angle Q$, isosceles triangle]

$\Rightarrow 2\angle Q + \angle P = 180^o$

$\Rightarrow 2\angle Q = 180 - 90^o$

$\Rightarrow \angle Q = 45^o$

Case Study – 1

13.

(i) In ΔEFD, $\tan 30^o = \frac{ED}{DF}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{DF}$

$\Rightarrow DF = h\sqrt{3}$ m

Now, In ΔGCE, $\tan 60^o = \frac{EC}{GC} = \frac{h+4}{DF}$

$\Rightarrow \sqrt{3} = \frac{h+4}{\sqrt{3}h}$

$\Rightarrow 3h = h + 4$

$\Rightarrow h = 2m$

(ii) Height of the balloon from the ground :-

= BC + CD + DE = 2 + 4 + 2 = 8m

Case Study – 2

14. No. of pairs of shoes in 1st, 2nd, 3rd row,..... are 3, 5, 7, ...

So, A.P. is formed with a = 3, d = 2

(i) $S_n = 120$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$120 = \frac{n}{2}\left[2(3) + (n-1)2\right]$$

$\Rightarrow$ $n^2 + 2n - 120 = 0$

$\Rightarrow$ $(n + 12)(n - 10) = 0$

$n = -12$ (neglect) or $n = 10$.

So, 10 rows required to put 120 pairs.

(ii) No. of pair in 17^{th} row :

$a_{17} = a + 16d$

$a_{17} = 35$

Also, no. of pair in 10^{th} row :

$a_{10} = a + 9d$

$a_{10} = 21$

$\therefore$ Req. diff = 35 – 21 = 14.